〃 为

〃 〃 又

很

(a,b $\sqrt{ab} < \frac{a-b}{\ln a - \ln b} < \frac{a+b}{2}$).

开， ，

〃 〃 $\{a_n\}$

$$(na_{n-1}-2)a_n = (2a_n - 1)a_{n-1}\ (n \ge 2),\ b_n = \frac{1}{a_n} - n(n \in N^*).$$

(1) $a_1 = 3$, $\{b_n\}$ ，

(2) $k \in N^*$, 很 $\frac{1}{a_k}, \frac{1}{a_{k+1}}, \frac{1}{a_{k+2}}$

$\{a_n\}$ 开，

$$\ln n + \frac{1}{2}a_n > \ln(n+1) - \frac{1}{2}a_{n+1}.$$

〃

$\ln n + \frac{1}{2}a_n > \ln(n+1) - \frac{1}{2}a_{n+1}$, $\frac{1}{2}(a_n + a_{n+1}) > \ln\frac{n+1}{n}$,

$a_n = \frac{1}{n}$, $\frac{1}{n} + \frac{1}{n+1} > 2\ln\frac{n+1}{n}$.

$t = \frac{n+1}{n}$, $\frac{1}{n} + \frac{1}{n+1} = t - 1 + \frac{t-1}{t} = t - \frac{1}{t}$, $t > 1$,

$t > 1$ $, t - \frac{1}{t} > 2\ln t$.

$f(x) = x - \frac{1}{x} - 2\ln x (x > 1)$, $f'(x) = 1 + \frac{1}{x^2} - \frac{2}{x} = \left(\frac{1}{x} - 1\right)^2 > 0$,

$f(x)$ $(1, +\infty)$

$f(x) > f(1) = 0$, $x - \frac{1}{x} > 2\ln x$,

$t > 1$, $t - \frac{1}{t} > 2\ln t$, 开很

受 $\frac{1}{n}+\frac{1}{n+1}>2\ln\frac{n+1}{n}$. $\frac{1}{2n}+\frac{1}{2(n+1)}>\ln(n+1)-\ln n$,

$$\frac{2n(n+1)}{n+(n+1)}<\frac{1}{\ln(n+1)-\ln n},$$

$$\sqrt{ab}<\frac{a-b}{\ln a-\ln b}<\frac{a+b}{2}\Rightarrow\sqrt{n(n+1)}<\frac{n+1-n}{\ln(n+1)-\ln n},$$

$$\frac{2n(n+1)}{n+n+1}<\sqrt{n(n+1)}\quad n+(n+1)>2\sqrt{n+(n+1)}.$$

很

〃 〃 $f(x)=2\ln x+\frac{1}{2}x^2-ax$ $a\in\mathbf{R}$

$a=3$ $f(x)$,

$f(x)$ $x=x_0$ $y=g(x)$ $y=f(x)-g(x)$ $(0\ \ +\infty)$

x_0 ,

$y=f(x)$

〃

$f(x)$ $T_1(x_1\ \ y_1)$ $T_2(x_2\ \ y_2)$

$0<x_1<x_2$ T_1 l_1 $y-f(x_1)=f'(x_1)(x-x_1)$

T_2 l_2 $y-f(x_2)=f'(x_2)(x-x_2)$

l_1 l_2 $\begin{cases}f'(x_1)=f'(x_2)\\ f(x_1)-x_1f'(x_1)=f(x_2)-x_2f'(x_2)\end{cases}$.

$$\begin{cases}\frac{2}{x_1}+x_1-a=\frac{2}{x_2}+x_2-a\\ 2\ln x_1+\frac{1}{2}x_1^2-ax_1-x_1\left(\frac{2}{x_1}+x_1-a\right)=2\ln x_2+\frac{1}{2}x_2^2-ax_2-x_2\left(\frac{2}{x_2}+x_2-a\right)\end{cases}.$$

很 $\begin{cases}x_1x_2=2\\ 2\ln x_1-\frac{1}{2}x_1^2=2\ln x_2-\frac{1}{2}x_2^2\end{cases}$.

x_2 很 $2\ln\frac{x_1^2}{2}+\frac{2}{x_1^2}-\frac{x_1^2}{2}=0$

$t=\frac{x_1^2}{2}$ $0<x_1<x_2$ $x_1x_2=2$ 很 $t\in(0\ \ 1)$

$p(t)=2\ln t+\frac{1}{t}-t$ $p'(t)=\frac{2}{t}-\frac{1}{t^2}-1=-\frac{(t-1)^2}{t^2}<0$

$p(t)$ $(0\ \ 1)$ $p(t)>p(1)=0$

开 $f(x)$

受 $f'(x_1)=f'(x_2)=\dfrac{f(x_1)-f(x_2)}{x_1-x_2}$,

$$\frac{2}{x_1}+x_1-a=\frac{2}{x_2}+x_2-a=\frac{2(\ln x_1-\ln x_2)+\frac{1}{2}(x_1{}^2-x_2{}^2)-a(x_1-x_2)}{x_1-x_2}$$

很 $\dfrac{2}{x_1}+x_1=\dfrac{2}{x_2}+x_2=\dfrac{2(\ln x_1-\ln x_2)}{x_1-x_2}+\dfrac{1}{2}(x_1+x_2)$,

$$\frac{2}{x_1}+x_1=\frac{2}{x_2}+x_2=\frac{2(\ln x_1-\ln x_2)}{x_1-x_2}+\frac{1}{2}(x_1+x_2)=t.$$

$\begin{cases}\dfrac{2}{x_1}+x_1=t\\ \dfrac{2}{x_2}+x_2=t\end{cases}$ $\begin{cases}x_1x_2=2\\ x_1+x_2=t\end{cases}$

$$\frac{2(\ln x_1-\ln x_2)}{x_1-x_2}+\frac{1}{2}(x_1+x_2)=x_1+x_2$$

很 $\dfrac{x_1-x_2}{\ln x_1-\ln x_2}=\dfrac{4}{x_1+x_2}$, $\sqrt{x_1x_2}<\dfrac{x_1-x_2}{\ln x_1-\ln x_2}<\dfrac{x_1+x_2}{2}$

$\sqrt{2}<\dfrac{4}{x_1+x_2}<\dfrac{x_1+x_2}{2}$ $\begin{cases}x_1+x_2<2\sqrt{2}\\ x_1+x_2>2\sqrt{2}\end{cases}$, .

〃 〃 $f(x)=\dfrac{a}{x}+\ln x(a\in R)$

$f(x)$ x_1,x_2.

$$p=x_1f'(x_1)+x_2f'(x_2)=1-\frac{a}{x_1}+1-\frac{a}{x_2}=2-\left(\frac{a}{x_1}+\frac{a}{x_2}\right).$$

$\begin{cases}\ln x_1+\dfrac{a}{x_1}=0\\ \ln x_2+\dfrac{a}{x_2}=0,\end{cases}$ $p=2+\ln(x_1x_2)$.

$x_1x_2>a^2$

$x_1x_2>a^2\Leftrightarrow\sqrt{x_1x_2}>a\Leftrightarrow\sqrt{x_1x_2}>\dfrac{\ln x_1-\ln x_2}{\dfrac{1}{x_2}-\dfrac{1}{x_1}}$ a

$$\Leftrightarrow\sqrt{x_1x_2}>\frac{\ln x_1-\ln x_2}{x_1-x_2}\cdot x_1x_2\Leftrightarrow\sqrt{x_1x_2}<\frac{x_1-x_2}{\ln x_1-\ln x_2}.$$

很

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参 二

$$\sqrt{ab}<\frac{a-b}{\ln a-\ln b}<\frac{a+b}{2}$$

$a>b>0$ $\sqrt{ab}<\frac{a-b}{\ln a-\ln b}\Leftrightarrow \ln\frac{a}{b}<\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}$

$\sqrt{\frac{a}{b}}=t(t>1)$ $f(t)=2\ln t-t+\frac{1}{t}(t>1)$

$f'(t)=\frac{2}{t}-1-\frac{1}{t^2}=\frac{-(t+1)^2}{t^2}<0$ $f(t)$ $(1,+\infty)$ $\because f(1)=0$

$t>1$ $f(t)=2\ln t-t+\frac{1}{t}<0$ $\ln\frac{a}{b}<\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}$

$\frac{a-b}{\ln a-\ln b}<\frac{a+b}{2}\Leftrightarrow \ln\frac{a}{b}>\frac{2(\frac{a}{b}-1)}{\frac{a}{b}+1}$

$\frac{a}{b}=t(t>1)$ $g(t)=\ln t-\frac{2(t-1)}{t+1}(t>1)$

$g'(t)=\frac{1}{t}-\frac{4}{(t+1)^2}=\frac{(t-1)^2}{t(t+1)^2}>0$ $g(t)$ $(1,+\infty)$ $\because g(1)=0$

$t>1$ $\ln t-\frac{2(t-1)}{t+1}>0$ $\frac{a-b}{\ln a-\ln b}<\frac{a+b}{2}$

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$a<\frac{2}{\frac{1}{a}+\frac{1}{b}}<\sqrt{ab}<\frac{a-b}{\ln a-\ln b}<\sqrt{\frac{a^2+b^2}{2}}<b\left(0<a<b\right)$, $x=\frac{b}{a}$,

$x\in(1,+\infty)$ $1<\frac{2x}{1+x}<\sqrt{x}<\frac{x-1}{\ln x}<\sqrt{\frac{1+x^2}{2}}<x$,

$x\in(0,1)$ $x<\frac{2x}{1+x}<\sqrt{x}<\frac{x-1}{\ln x}<\sqrt{\frac{1+x^2}{2}}<1$,

e^{x_1} a, e^{x_2} b, 很 开 $e^{\frac{x_1+x_2}{2}}<\frac{e^{x_1}-e^{x_2}}{x_1-x_2}<\frac{e^{x_1}+e^{x_2}}{2}$,

$n=a, n+1=b$,

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